

To find time constant:

Differentiate $C = C_0 e^{-kt}$.

$$\ln C = \ln C_0 - kt.$$

$$\frac{dC}{dt} \times \frac{1}{C} = -k.$$

$$\frac{dC}{dt} = -kC.$$

This gives gradient at any point on $C = C_0 e^{-kt}$.

The rate constant line equation is now:

$$y = mx + c.$$

$$C = -kC_0 t + b.$$

When $C = C_0, t = 0 \therefore$

$$C_0 = b.$$

$$C = C_0 - kC_0 t$$

when $C = 0$:

$$0 = C_0 - kC_0 t$$

$$-C_0 = -kC_0 t$$

$$t = \frac{1}{k} \rightarrow \text{now input this in to } C = C_0 e^{-kt}$$

$$C = C_0 e^{-kt}$$

$$C = C_0 e^{-k(\frac{1}{k})}$$

$$C = \frac{C_0}{e}$$

When $C = \frac{C_0}{e}$, $t = T$:

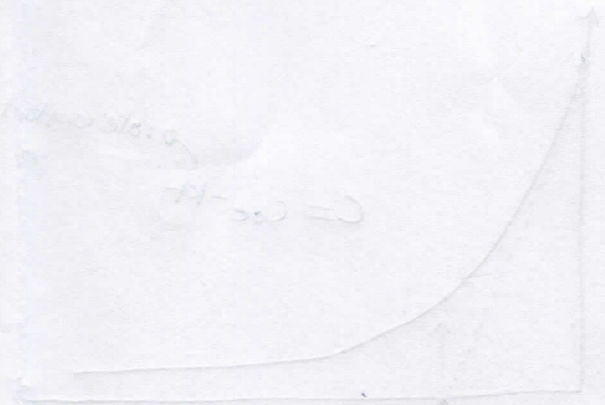
$$\frac{C_0}{e} = C_0 e^{-kT}$$

$$e^{-1} = e^{-kT}$$

$$-1 = -kT$$

$$T = \frac{1}{k}$$

↑
time constant.



Time constant = Resistance \times Compliance

~~Volume~~ ~~Pressure~~

$$= \frac{\text{Pressure}}{\text{Flow}} \times \frac{\text{Volume}}{\text{Pressure}} = \frac{\text{Volume}}{\text{Flow}}$$

~~Compliance~~

$$\frac{C_0}{2} = C_0 e^{-k t_{1/2}}$$

$$-\ln 2 = -k t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{k}$$

$$k = \frac{1}{T} \therefore t_{1/2} = \ln 2 \times T$$